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Bosonization, coherent states and semiclassical quantum Hall skyrmions

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Abstract
We bosonize (2 + 1)-dimensional fermionic theory using coherent states. The gauge-invariant subspace of boson–Chern–Simons Hilbert space is mapped to fermionic Hilbert space. This subspace is then equipped with a coherent state basis. These coherent states are labelled by a dynamic spinor field. The label manifold could be assigned a physical meaning in terms of density and spin density. A path-integral representation of the evolution operator in terms of these physical variables is given. The corresponding classical theory when restricted to LLL is described by spin fluctuations alone and is found to be the NLSM with Hopf term. The formalism developed here is suitable to study quantum Hall skyrmions semiclassically and/or beyond the hydrodynamic limit. The effects of Landau level mixing or the presence of slowly varying external fields can also be easily incorporated.

1. Introduction

Bosonization provides a dual description for fermionic systems and exposes them to complementary enquiry. It is not only applied as a tool to probe these fermionic systems but continues to be an active topic of research. Bosonization in 1 + 1 dimensions has been thoroughly and exhaustively studied. Even in 2 + 1 dimensions there are numerous works and it is implemented in various different approaches [1–4].

In 2 + 1 dimensions the transmutation of statistics can be induced by the Chern–Simons gauge field [5]. But this gauge field is not a dynamical field and hence introduces a redundancy into the description of the fermionic system. In the Hilbert space that describes both the bosons and the Chern–Simons gauge field, the gauge-invariant subspace is sufficient to describe the fermionic system. We provide a coherent state basis to this subspace and label the coherent states by a dynamic spinor field.

Coherent states have the advantage of providing a correspondence between the classical and the quantum dynamics. It also enables us to implement quantum constraints as conditions on the classical manifold that labels them. For example, in quantum Hall systems the typical energy scales involved make the dynamics predominantly to take place in the lowest Landau level (LLL). Such quantum conditions can be implemented easily in a coherent state basis.

Bosonization of (2 + 1)-dimensional quantum Hall systems when restricted to LLL is known to result in a (1 + 1)-dimensional bosonic theory [6]. One of the advantages of that bosonic theory is that tools of low-dimensional theories aid in furthering insights, while a major disadvantage is the difficulty in incorporating the effects of Landau level mixing. Moreover the approach used in [6] is not amenable to generalize bosonization beyond LLL.

Spinless quantum Hall systems, without any LLL restriction, have been bosonized using the Chern–Simons gauge field, and successfully applied to study the fractional quantum Hall effect [7]: Laughlin ground states, vortex excitations, etc. Though the approach described in [7] can easily be extended to bosonize electrons with spin, the resulting bosonic Hilbert space has unphysical states. Furthermore, it has difficulty in identifying a correspondence between the classical solutions and the quantum states.

Systems that exhibit Fermi liquid behaviour have also been bosonized in higher dimensions within the long-wavelength limit [8, 9]. The low-energy particle–hole excitations of such systems are associated with small deformations of the Fermi surface and furthermore are depicted by coherent states [9]. Within the same approximation, even LLL systems have been provided with a bosonic algebra to describe the fermion bilinear operators like charge and spin density [10]. The coherent states of these bosons are then made use of to represent the skyrmionic excitations at filling fraction $\nu = 1$.

Apart from providing a new bosonization scheme and a coherent state basis labelled only by dynamic variables, one of the motivations of this work is to develop a suitable formalism for semiclassical investigation of systems with quantum constraints. In particular, we systematically derive the effective theory that is used for describing the quantum Hall skyrmions, namely the nonlinear sigma model (NLSM), at filling fractions $\nu = 1/m$, where m is any odd integer. What is the limit when this classical theory becomes exact? What are the corrections to this NLSM when this limit is not yet reached? What is the effect of Landau level mixing?

Quantum Hall physics has given rise to an intensely active area where various complementary approaches are pursued to perceive its intricacies (see [11] and references therein). Attaching fluxes, for example, to fermions instead of bosons [11, 12] has systematically led to effective theories at Jain filling fractions. In fact, quantum Hall systems exhibit such a rich variety of excitations, like, for example, various skyrmions, that it still continues to captivate both theorists and experimentalists [13, 14]. Nevertheless, we shall not attempt to mention here the enormous list of aspects and objects this fascinating subject offers.

The layout of this paper is as follows. In section 2, we construct a coherent state basis for the Hilbert space of the bosons and the Chern–Simons gauge fields. We then construct a representation for the anticommuting electron operators in terms of the bosonic fields and provide an explicit mapping between the states and observables of electronic Hilbert space and the states and operators of the gauge-invariant sector of the composite boson theory.

In section 3, we describe the physical Hilbert space which is the gauge-invariant subspace of the composite boson Hilbert space. The coherent states are projected onto this subspace and their wavefunctions calculated. The explicit form of the wavefunctions is made use of in parametrizing the projected coherent states. The LLL condition is also found to be equivalent to an analyticity condition on this parameter space. The matrix elements of the observables between the projected coherent states are expressed as correlation functions of a field theory, and evaluated in the hydrodynamic limit. We also discuss the relation between the charge and the topological charge densities.

In section 4, we show that the projected coherent states satisfy the properties of generalized coherent states [15]. The limit where a classical description is admitted is dealt with in section 5. We finally conclude in section 6 with a summary, then apply the formalism to justify the nonlinear sigma model description of skyrmions in quantum Hall systems, and comment about other possible applications.

2. Composite boson theory and coherent states

In this section, we bosonize the fermionic system in 2 + 1 dimensions, wherein the electrons are described by the composite objects composed of bosons and fluxes of the Chern–Simons field. We first define the Hilbert space of the composite bosons and construct a coherent state basis for it. We then give a representation of the electron field operators and provide an explicit mapping between the states and observables of the fermionic system and the gauge-invariant states and operators of the bosonic theory. The procedure of attaching fluxes to bosons and the construction of electron operators is quite transparent in terms of these coherent states.

2.1. The composite boson Hilbert space

The bosonic degrees of freedom are described by the spinor-field operators, $\widehat{\varphi}_\sigma(x)$ and $\widehat{\varphi}^\dagger_\sigma(x)$, and the Chern–Simons gauge-field operators, $\widehat{a}_i(x)$. Here both the indices σ and i run over 1 and 2, and x denotes a point in two-dimensional space. The $\{\widehat{\varphi}\}$ operators act on the Hilbert space $\mathcal{H}_{\rm B}$ and satisfy the canonical commutation relations

$$\begin{aligned}[\widehat{\varphi}_\sigma(x), \widehat{\varphi}^\dagger_{\sigma'}(y)] &= \delta_{\sigma\sigma'}\delta(x-y),\\ [\widehat{\varphi}_\sigma(x), \widehat{\varphi}_{\sigma'}(y)] &= [\widehat{\varphi}^\dagger_\sigma(x), \widehat{\varphi}^\dagger_{\sigma'}(y)] = 0.\end{aligned} \tag{2.1}$$

The Chern–Simons gauge field acts on the Hilbert space $\mathcal{H}_{\rm CS}$ and satisfies

$$\left[\widehat{a}_i(x), \widehat{a}_j(y)\right] = \kappa^{-1/2}\epsilon_{ij}\delta(x-y), \tag{2.2}$$

where κ is an arbitrary parameter as yet, but shall be restricted later while constructing a fermionic representation in this composite bosonic Hilbert space. It is convenient to write the Chern–Simons gauge fields in terms of the complex fields, $\widehat{a}(x)$ and $\widehat{a}^\dagger(x)$, where

$$\begin{aligned}\widehat{a}(x) &:= \sqrt{\frac{\kappa}{2}}(\widehat{a}_2(x) + \mathrm{i}\widehat{a}_1(x))\\ \widehat{a}^\dagger(x) &:= \sqrt{\frac{\kappa}{2}}(\widehat{a}_2(x) - \mathrm{i}\widehat{a}_1(x))\end{aligned} \tag{2.3}$$

which then satisfy the commutation relations

$$\left[\widehat{a}(x), \widehat{a}^\dagger(y)\right] = \delta(x-y). \tag{2.4}$$

The Hilbert space of the composite boson theory is now defined as the direct product of these two bosonic spaces and is denoted by

$$\mathcal{H}_{\rm CB} = \mathcal{H}_{\rm B} \otimes \mathcal{H}_{\rm CS}. \tag{2.5}$$

The gauge-invariant sector of this space, $\mathcal{H}_{\rm phy} \subset \mathcal{H}_{\rm CB}$, describes the states of the physical theory and is the set of all states that respect the Chern–Simons Gauss law constraint

$$\widehat{G}(x)|\psi\rangle_{\rm phy} = 0, \tag{2.6}$$

where $\hat{G}(x)$ is the generator of the gauge transformation and is given by

$$\widehat{G}(x) = \kappa\nabla \times \widehat{\vec{a}}(x) - e\widehat{\varphi}^\dagger(x)\widehat{\varphi}(x). \tag{2.7}$$

Henceforth, the gauge-invariant operators, namely the operators that commute with $\widehat{G}(x)$ and act on $\mathcal{H}_{\rm phy}$, will also be referred to as physical observables.

2.2. Coherent state basis

We now construct the coherent state basis for the composite boson Hilbert space. Since $\mathcal{H}_{CB}$ is the direct product of two spaces, both of which contain the canonical harmonic oscillator structure, the most natural construction is to take the product of the field coherent states of $\mathcal{H}_B$ and $\mathcal{H}_{CS}$.

Field coherent states are defined by many equivalent definitions [16], one of which is through the displacement operators defined as

$$D(a) := \exp\left(\int_x a(x)\widehat{a}^\dagger(x) - \bar{a}(x)\widehat{a}(x)\right), \tag{2.8}$$

and

$$U(\varphi) := \exp\left(\int_x \varphi_\sigma(x)\widehat{\varphi}_\sigma(x)^\dagger - \bar{\varphi}_\sigma(x)\widehat{\varphi}_\sigma(x)\right), \tag{2.9}$$

where $a(x)$ and $\{\varphi_\sigma(x)\}$ are complex fields, and $\bar{a}(x)$ and $\{\bar{\varphi}_\sigma(x)\}$ denote the corresponding complex conjugates. These operators are called displacement operators for they possess the property $D(-a)\widehat{a}(x)D(a) = \widehat{a}(x) + a(x)$ and $U(-\varphi)\widehat{\varphi}_\sigma(x)U(\varphi) = \widehat{\varphi}_\sigma(x) + \varphi_\sigma(x)$.

The coherent states $|a, \varphi\rangle$ are defined by acting the displacement operators on the vacuum $|0\rangle$. Hence they are parametrized by the gauge field $a(x)$ and by the spinor field $\varphi(x)$ with components $\{\varphi_\sigma(x)\}$, and are given as

$$|a, \varphi\rangle := U(\varphi)D(a)|0\rangle, \tag{2.10}$$

where

$$\widehat{a}(x)|0\rangle = \widehat{\varphi}_\sigma(x)|0\rangle = 0. \tag{2.11}$$

These states can be interpreted as Gaussian wavepackets peaked around the classical field configuration $\{a(x), \varphi(x)\}$. They satisfy the three standard properties of coherent states [15], namely:

(1) Resolution of identity:

$$\int \mathcal{D}[a, \varphi]|a, \varphi\rangle\langle a, \varphi| = I, \tag{2.12}$$

where the measure $\mathcal{D}[a, \varphi] = \prod_x (2\pi \mathrm{i})^{-3} \mathrm{d}a(x)\,\mathrm{d}\bar{a}(x) \prod_\sigma \mathrm{d}\varphi_\sigma(x)\,\mathrm{d}\bar{\varphi}_\sigma(x)$.

(2) Continuity of overlaps:

$$\langle a_1, \varphi_1|a_2, \varphi_2\rangle = \exp\left(-\mathrm{i}\frac{\kappa}{2}\int_x \vec{a}_1 \times \vec{a}_2 - \frac{\kappa}{4}\int_x |\vec{a}_1 - \vec{a}_2|^2\right) \times \exp\left(\tfrac{1}{2}\int_{x,\sigma} (\bar{\varphi}_{1\sigma}\varphi_{2\sigma} - \varphi_{1\sigma}\bar{\varphi}_{2\sigma} - |\varphi_{1\sigma} - \varphi_{2\sigma}|^2)\right), \tag{2.13}$$

where, to keep the notation compact, $\vec{a} = \vec{a}(x)$ and $\varphi = \varphi(x)$, are used. Here the components of the vector $\vec{a} \equiv (a_1, a_2) = \sqrt{2/\kappa}(\mathrm{Im}(a), \mathrm{Re}(a))$, are defined as the imaginary and real part of the complex field a.

(3) Expectation values of observables:

$$\langle a, \varphi| : O(\widehat{a}, \widehat{a}^\dagger, \widehat{\varphi}, \widehat{\varphi}^\dagger) : |a, \varphi\rangle = O(a, \bar{a}, \varphi, \varphi^\dagger)\,\langle a, \varphi|a, \varphi\rangle. \tag{2.14}$$

Note that these coherent states are not gauge-invariant and transform as

$$|a, \varphi\rangle \to \mathrm{e}^{\mathrm{i}\int \widehat{G}(x)\Omega(x)}\,|a, \varphi\rangle = \mathrm{e}^{\mathrm{i}(\kappa/2)\int_x \bar{a}(x)\times\nabla\Omega(x)}\,\left|a - \nabla\Omega, \varphi\mathrm{e}^{-\mathrm{i}e\Omega}\right\rangle, \tag{2.15}$$

under gauge transformations.

We now construct a projection operator that projects any state into the gauge-invariant subspace, $\mathcal{H}_{phy}$. Consider the operator

$$P := \frac{1}{V_G}\int_\Omega \exp\left(\mathrm{i}\int_x \Omega(x)\widehat{G}(x)\right) \tag{2.16}$$

where $\widehat{G}(x)$ is the generator of gauge transformations, as given in equation (2.7), $\Omega(x)$ is a real-valued smooth function of compact support and $V_G = \int_\Omega$ is the volume of the gauge group. It is easy to see that this operator has the property $P^2 = P$. Shifting the integration variable Ω by β in the above expression for the projection operator P gives

$$\mathrm{e}^{\mathrm{i}\int_x \beta(x)\widehat{G}(x)} P = P, \tag{2.17}$$

and then taking the limit $\beta \to 0$ leads to

$$\widehat{G}(x)P = 0 \Rightarrow \widehat{G}(x)P|\psi\rangle = 0. \tag{2.18}$$

Hence the operator P projects any state into $\mathcal{H}_{phy}$.

The above three properties (2.12)–(2.14) and the projection operator defined in equation (2.16) can be used to derive the path-integral representation of the gauge-invariant evolution operator. We obtain this path integral, as detailed in appendix A, to be

$$Z = \int \mathcal{D}[a_0(x,t)]\mathcal{D}[a_i(x,t)]\mathcal{D}[\varphi(x,t)]\mathrm{e}^{\mathrm{i}\int \mathrm{d}t\,\mathrm{d}^2x\,\mathcal{L}(x,t)}, \tag{2.19}$$

with the standard Lagrangian of a couplet of matter fields coupled to the Chern–Simons gauge field, given by

$$\mathcal{L}(x,t) = -\frac{\kappa}{2}\epsilon_{\mu\nu\lambda}a^\mu(x,t)\partial^\nu a^\lambda(x,t) + \mathrm{i}\varphi^\dagger(x,t)\dot{\varphi}(x,t) + ea_0(x,t)\varphi^\dagger(x,t)\varphi(x,t) - \mathcal{H}(a(x,t), \varphi(x,t)), \tag{2.20}$$

where $\mathcal{H}$ is the Hamiltonian density. This confirms the equivalence of our formalism and the standard Lagrangian formalism. In other words, this Chern–Simons classical theory, which is described by a pair of conjugate variables and a first-class constraint, upon quantization would yield the quantum theory that we proposed here.

2.3. Bosonization

We now construct gauge-invariant anticommuting operators that create and annihilate flux-carrying bosons. These operators satisfy the fermionic canonical anticommutation relations and hence are used for representing the electron creation and annihilation operators in $\mathcal{H}_{CB}$. With this electron representation we provide a map from the gauge-invariant sector of the composite boson Hilbert space, $\mathcal{H}_{phy}$, onto the Hilbert space of the electronic system, $\mathcal{H}_{el}$. The inverse mapping then gives a representation of the observables of the electronic system as gauge-invariant operators in $\mathcal{H}_{CB}$.

We define the operator $c_\sigma^\dagger(x)$ as

$$c_\sigma^\dagger(x) := D(x)\hat{\varphi}_\sigma^\dagger(x)K(x), \tag{2.21}$$

where, to simplify the notation, $D(x)$ is used for $D(\alpha_x^v)$. Here α_x^v is the classical configuration of a vortex with a delta function flux density at the point x, and obeys the relation

$$\kappa\nabla\times\vec{\alpha}_x^v(z) = e\delta(z-x). \tag{2.22}$$

Hence $D(x)$ can be interpreted as an operator that creates a Gaussian wavepacket peaked around this classical vortex configuration. When $c^\dagger(x)$ acts on a state, it amounts to creating a bosonic particle at x by $\hat{\varphi}_\sigma^\dagger(x)$, and then attaching to it a Chern–Simons flux by $D(x)$. The operator $K(x)$ gives the Aharanov–Bohm phase corresponding to all the other particles already present in the state. It is defined as

$$K(x) := \exp\left(\mathrm{i}m\int_z \theta(x-z)\hat{\varphi}^\dagger(z)\hat{\varphi}(z)\right) \tag{2.23}$$

where m is an odd integer and $\theta(x)$ is the angle the vector, x, makes with the x axis. Though $K(x)$ is a nonlocal operator the local observables of the $\mathcal{H}_{\mathrm{el}}$ get mapped to local operators of $\mathcal{H}_{\mathrm{phy}}$.

The annihilation operators $c_\sigma(x)$ are defined as the Hermitian conjugates of $c_\sigma^\dagger(x)$. Using the commutation relations given in equations (2.1) and (2.2), it can be verified that the following canonical anticommutation relations hold good when $\kappa = \mathrm{e}^2/2\pi m$:

$$\begin{gathered}\{c_\sigma(x), c_{\sigma'}^\dagger(y)\} = \delta_{\sigma\sigma'}\delta(x-y),\\ \{c_\sigma(x), c_{\sigma'}(y)\} = \{c_\sigma^\dagger(x), c_{\sigma'}^\dagger(y)\} = 0.\end{gathered} \tag{2.24}$$

Hence $c_\sigma^\dagger(x)$ and $c_\sigma(x)$ provide a representation of the electron creation and annihilation operators in $\mathcal{H}_{\mathrm{CB}}$. Since under gauge transformation

$$\begin{aligned}\widehat{\varphi}_\sigma(x) &\to \mathrm{e}^{\mathrm{i}\,\mathrm{e}\Omega(x)}\widehat{\varphi}_\sigma(x)\,, \qquad & \widehat{\varphi}_\sigma^\dagger(x) &\to \mathrm{e}^{-\mathrm{i}\,\mathrm{e}\Omega(x)}\widehat{\varphi}_\sigma^\dagger(x)\\ a_i(x) &\to a_i(x) + \partial_i\Omega(x), & D(x) &\to \mathrm{e}^{\mathrm{i}\,\mathrm{e}\Omega(x)}D(x),\end{aligned} \tag{2.25}$$

we see that $c_\sigma(x)$ and $c_\sigma^\dagger(x)$ are gauge-invariant.

We are now in a position to map $\mathcal{H}_{\mathrm{phy}}$ into $\mathcal{H}_{\mathrm{el}}$. We map the vacuum state of $\mathcal{H}_{\mathrm{CB}}$ projected to $\mathcal{H}_{\mathrm{phy}}$ to the zero-electron sector state $|0\rangle_{\mathrm{el}}$ of the electronic Hilbert space:

$$P|0\rangle \to |0\rangle_{\mathrm{el}}. \tag{2.26}$$

Since $c_\sigma(x)$ are gauge-invariant, and hence commute with P, it is easy to see that

$$c_\sigma(x)P|0\rangle = 0. \tag{2.27}$$

The state with N electrons at $(x_1, x_2, \ldots x_N)$ with spins $(\sigma_1, \sigma_2, \ldots, \sigma_N)$, denoted $|\{x_a, \sigma_a\}_N\rangle$, is then given a correspondence with

$$\prod_{a=1}^{N} c_{\sigma_a}^\dagger(x_a)P|0\rangle \to |\{x_a, \sigma_a\}_N\rangle. \tag{2.28}$$

Since the states on the right-hand side of equations (2.26) and (2.28) form a basis for $\mathcal{H}_{\mathrm{el}}$, these equations specify the explicit mapping of $\mathcal{H}_{\mathrm{phy}}$ into $\mathcal{H}_{\mathrm{el}}$.

It is now easy to identity the operators in $\mathcal{H}_{\mathrm{phy}}$ that correspond to the physical observables. The density and the spin density operators are

$$\widehat{\rho}(x) = c_\sigma^\dagger(x)c_\sigma(x) = \widehat{\varphi}_\sigma^\dagger(x)\widehat{\varphi}_\sigma(x), \tag{2.29}$$

and

$$\widehat{S}^\mu(x) = \tfrac{1}{2}c_\sigma^\dagger(x)\tau_{\sigma\sigma'}^\mu c_{\sigma'}(x) = \tfrac{1}{2}\widehat{\varphi}_\sigma^\dagger(x)\tau_{\sigma\sigma'}^\mu\widehat{\varphi}_\sigma(x), \tag{2.30}$$

respectively, where $\{\tau^\mu\}$ are Pauli matrices. The current density is given by

$$\begin{aligned}\widehat{J}_i(x) &= \tfrac{1}{2}c_\sigma^\dagger(x)\left(-\mathrm{i}\partial_i - eA_i(x)\right)c_\sigma(x) + \text{h.c.}\\ &= \tfrac{1}{2}\widehat{\varphi}_\sigma^\dagger(x)\left(-\mathrm{i}\partial_i - ea_i(x) - eA_i(x)\right)\widehat{\varphi}_\sigma(x) + \text{h.c.},\end{aligned} \tag{2.31}$$

where an additional term, $-\widehat{\varphi}_\sigma^\dagger(x)\widehat{\varphi}_\sigma(x)\int_z \alpha_{xi}^v(z)\widehat{G}(z)$, has been dropped out, since its action on physical states is zero. Similarly, the kinetic energy density is given by

$$\widehat{T}(x) = \frac{1}{2m}\widehat{\varphi}_\sigma^\dagger(x)\left(-\mathrm{i}\partial_i - ea_i(x) - eA_i(x)\right)^2\widehat{\varphi}_\sigma(x). \tag{2.32}$$

Note that c_σ and $c_\sigma^\dagger$ are composite operators and hence ill-defined unless they are renormalized. In fact, this inverse map takes the operators of $\mathcal{H}_{\mathrm{el}}$ to the bare operators of $\mathcal{H}_{\mathrm{phy}}$ which need to be renormalized. The correspondence between the states of $\mathcal{H}_{\mathrm{el}}$ and $\mathcal{H}_{\mathrm{phy}}$ though remains unaffected by the field renormalization of c_σ and $c_\sigma^\dagger$.

It should be remarked that a particularly interesting scheme was adapted by Lüscher to bosonize a fermionic field coupled to the Chern–Simons gauge field [1]. The construction employed in [1] has similar elements as our construction here. One of the differences being, there the quantum theory has $\{a_i\}$ fields and their conjugate momenta as the fundamental fields, while here $\{a_i\}$ fields form a conjugate pair. Another difference, and a novel angle, to our construction comes from employing coherent states to bosonize. This scheme has an added advantage, apart from enabling the fermionic theory to subjugate to semiclassical methods, of implementing the quantum constraints as conditions on the classical manifold that label the coherent states.

3. Physical Hilbert space

In this section, we study the properties of the coherent states when projected into $\mathcal{H}_{\mathrm{phy}}$. These projected states span the physical Hilbert space. We compute their wavefunctions and parametrize them by a single complex spinor field $W_\sigma(x)$. The LLL condition is found to be equivalent to an analyticity condition on the parameters. The expectation values of observables are evaluated in the hydrodynamic limit. We also discuss the relation between the charge density and the topological charge density.

3.1. Projected coherent states: wavefunctions

The coherent states of $\mathcal{H}_{\rm CB}$ form a basis, and hence these states when projected to $\mathcal{H}_{\rm phy}$ will span the physical Hilbert space. We shall see in the next section that they also form a coherent state basis of $\mathcal{H}_{\rm phy}$. We now evaluate the wavefunctions of these projected coherent states:

$$|a,\varphi\rangle_{\rm p} := P|a,\varphi\rangle. \qquad (3.1)$$

The coherent states are not eigenstates of the number operator. This property is unaffected by the P operation and hence the projected states also have a non-zero overlap with states containing any number of particles. The wavefunction in the N-particle sector is obtained by taking the overlap with the position basis states that are given in equation (2.28):

$$\psi_N(\{x_a,\sigma_a\}) = \langle\{x_a,\sigma_a\}_N|a,\varphi\rangle_{\rm p}. \qquad (3.2)$$

The explicit form of these wavefunctions, as detailed in appendix B, is given as

$$\psi_N(\{x_a,\sigma_a\}) = C\left(\prod_{a=1}^{N}\varphi_{\sigma_a}(x_a){\rm e}^{{\rm e}(\Omega_{\rm T}(x_a)-\widetilde{\Omega}_{\rm T}(x_a)-{\rm i}\Omega_{\rm L}(x_a))}\right) \times \psi_{\rm L}(\{x_a\}), \qquad (3.3)$$

where $C = C[a,\varphi]$ (as given in appendix B) is independent of the coordinates $\{x_a,\sigma_a\}$, whose amplitude depends on a and whose modulus depends both on $\varphi^\dagger\varphi$ and $\Omega_{\rm T}$. The gauge field a is decomposed into transverse and longitudinal components:

$$a_i(x) = \epsilon_{ij}\partial_j\Omega_{\rm T}(x) + \partial_i\Omega_{\rm L}(x), \qquad (3.4)$$

and $\psi_{\rm L}(\{x_a\})$ denotes the Laughlin wavefunction:

$$\psi_{\rm L}(\{x_a\}) = \prod_{a>b}(z_a - z_b)^m \exp\left(-\frac{1}{4l_B^2}\sum_a |x_a|^2\right). \qquad (3.5)$$

The function $e\widetilde{\Omega}_{\rm T}(x) := -|x|^2/4l_B^2$, is just the Gaussian part of the Laughlin wavefunction and l_B denotes the magnetic length.

Note that if we choose $\varphi_\sigma(x)$ to be a constant and $\Omega_{\rm T}(x) = \widetilde{\Omega}_{\rm T}(x)$, or in other words a uniform flux $\nabla\times\vec{a} = (|e|/e)B$, then the wavefunction reduces to the Laughlin wavefunction with density $\overline{\rho} = 1/2\pi m l_B^2$. Hence the coherent wavepacket, peaked around a constant φ and around a uniform flux equal to the external magnetic flux, when projected to $\mathcal{H}_{\rm phy}$ corresponds to the Laughlin state.

The evaluation of these wavefunctions also elucidates an interesting property of the projection operator, as shown in appendix C, namely it provides a correspondence between the coherent states and vertex operators of a 2D bosonic theory with a background charge. This connection between the states of quantum Hall systems and the vertex operators has been well investigated by many authors [17–19].

3.2. Parametrization and LLL condition

Apart from an overall factor that only affects the norm, the wavefunction in equation (3.3) depends on the parameters a and φ through a spinor field $W_\sigma(x)$ defined as

$$W_\sigma(x) := \varphi_\sigma(x)\exp({\rm e}(\Omega_{\rm T}(x) - \widetilde{\Omega}_{\rm T}(x) - {\rm i}\Omega_{\rm L}(x))). \qquad (3.6)$$

Under gauge transformations

$$\begin{aligned}\Omega_{\rm T}(x) &\to \Omega_{\rm T}(x)\\ \Omega_{\rm L}(x) &\to \Omega_{\rm L}(x) + \Omega(x)\\ \varphi_\sigma(x) &\to \varphi_\sigma(x){\rm e}^{{\rm i}e\Omega(x)},\end{aligned} \qquad (3.7)$$

$W_\sigma(x)$ remains invariant and (as it should be) so does the wavefunction. From the above transformation, it is easy to see that the fields a and φ together have six real field components. The gauge invariance of the wavefunctions reduces this number by one. There is another local invariance of W, when

$$\begin{aligned}\Omega_{\rm T}(x) &\to \Omega_{\rm T}(x) + \chi(x)\\ \Omega_{\rm L}(x) &\to \Omega_{\rm L}(x)\\ \varphi_\sigma(x) &\to \varphi_\sigma(x){\rm e}^{-e\chi(x)}.\end{aligned} \qquad (3.8)$$

Under this transformation only the norm of the state changes and so the physical state remains the same. Clearly this transformation is not unitarily implemented in $\mathcal{H}_{\rm CB}$; nonetheless it reduces the number of independent real fields that parametrize the states to four, namely the components of the spinor field W. Thus we can define the normalized projected coherent states that are parametrized by W as

$$|W\rangle = \frac{1}{\sqrt{\mathcal{N}}}|\alpha,\varphi\rangle_{\rm p}, \qquad (3.9)$$

where the norm $\mathcal{N} = {}_{\rm p}\langle\alpha,\varphi|\alpha,\varphi\rangle_{\rm p}$.

From equations (3.3) and (3.6), we see that the these wavefunctions belong to LLL if W is analytic:

$$\partial_{\bar{z}}W_\sigma(x) = 0. \qquad (3.10)$$

Thus the LLL condition is easily implemented in this formalism as it is equivalent to an analyticity condition on the labelling parameter W of the projected coherent states. These projected coherent state wavefunctions when restricted to LLL have the same form as considered by Ezawa [20].

3.3. Observables

We now compute the expectation values of gauge-invariant operators $\widehat{O}$ in the projected coherent states, denoted by

$$O(W) = \langle W|\widehat{O}|W\rangle. \qquad (3.11)$$

In fact, these diagonal elements are sufficient to specify the operator since the coherent states form an over-complete basis.

The norm of the projected state and the expectation values of observables in this state can be written as the partition function and correlation functions of a field theory, respectively. Here this correspondence is obtained by making, in each N-particle sector, the transformation

$$\{x_i\} \to \widetilde{\rho}(x) = \sum_{i=1}^{N}\delta(x - x_i), \qquad (3.12)$$

and functions $F(\{x_i\}) \to \mathcal{F}(\widetilde{\rho})$, where

$$\frac{1}{N!}\prod_{i=1}^{N}\int {\rm d}x_i\, F(\{x_i\}) = \int \mathcal{D}[\widetilde{\rho}]J[\widetilde{\rho}]\mathcal{F}(\widetilde{\rho}), \qquad (3.13)$$

and $J[\widetilde{\rho}]$ is the Jacobian of the transformation.

Thus the norm of the state $|a, \varphi\rangle_{\rm p}$ becomes

$$\mathcal{N}[W, \Omega_{\rm T}] = \sum_N \frac{1}{N!} \prod_{a=1}^N \int_{x_a} \sum_{\sigma_a} |\psi_N(\{x_a, \sigma_a\})|^2 = |C|^2 \mathcal{Z}[W^\dagger W], \quad (3.14)$$

where the partition function, for any given function $\Lambda(x)$, is

$$\mathcal{Z}[\Lambda] = \int \mathcal{D}[\widetilde{\rho}] \exp\left(-\mathcal{S}(\widetilde{\rho}; \Lambda)\right), \quad (3.15)$$

with an action

$$\mathcal{S}(\widetilde{\rho}; \Lambda) = -\ln J[\widetilde{\rho}] - \int_x \widetilde{\rho}(x)(\ln \Lambda(x) + 2e\widetilde{\Omega}_{\rm T}(x) + 2\pi m \nabla^{-2} \widetilde{\rho}(x)). \quad (3.16)$$

Expectation values of the operators can now be obtained by following similar calculational steps as taken in evaluating the norm. We now write down density, spin density, current density and kinetic energy density as correlation functions of this field theory.

In the N-particle sector the action of density operator $\widehat{\rho}(x)$ on the wavefunction has the simple form, $\sum_{i=1}^N \delta(x - \hat{x}_i)\psi_N = \widetilde{\rho}(x)\psi_N$, and hence we get

$$\rho(x) := \langle W|\widehat{\rho}(x)|W\rangle = \langle \widetilde{\rho}(x)\rangle_{\mathcal{S}} \quad (3.17)$$

where $\langle \cdots \rangle_{\mathcal{S}}$ denotes

$$\langle \cdots \rangle_{\mathcal{S}} := \frac{1}{\mathcal{Z}[W^\dagger W]} \int \mathcal{D}[\widetilde{\rho}] \cdots {\rm e}^{-\mathcal{S}(\widetilde{\rho}; W^\dagger W)}. \quad (3.18)$$

Similarly, for the spin density operator $\widehat{S}^\mu(x)$ in the N-particle sector, we get

$$\int_{\{x_a,\sigma_a\}} \overline{\psi}_N \sum_{i=1}^N \delta(x - \hat{x}_i) \frac{\tau_a^\mu}{2} \psi_N = \frac{1}{2} \frac{W^\dagger(x)\tau^\mu W(x)}{W^\dagger(x)W(x)} \int_{\{x_a,\sigma_a\}} \widetilde{\rho}(x)\, |\psi_N|^2, \quad (3.19)$$

and hence

$$S^\mu(x) := \langle W|\widehat{S}^\mu(x)|W\rangle = \tfrac{1}{2}\rho(x) Z^\dagger(x)\tau^\mu Z(x), \quad (3.20)$$

where the normalized spinor

$$Z_\sigma(x) := \frac{W_\sigma(x)}{\sqrt{W^\dagger(x)W(x)}}. \quad (3.21)$$

The current density is calculated to be

$$J_i(x) := \langle W|\widehat{J}_i(x)|W\rangle = \rho(x)\left(L_i^3(x) - eA_i(x)\right) + m\left\langle \widetilde{\rho}(x)\partial_{x_i} \int_y \theta(x-y)\widetilde{\rho}(y)\right\rangle_{\mathcal{S}}, \quad (3.22)$$

where $L_i^3 := \frac{1}{2{\rm i}}(Z^\dagger \partial_i Z - h.c)$.

The kinetic energy density in the N-particle sector is given by

$$\int_{\{x_a,\sigma_a\}} \sum_{i=1}^N |D_i\psi_N|^2 = \omega_{\rm c} \frac{\partial_z W^\dagger(x)\partial_{\bar{z}} W(x)}{W^\dagger(x)W(x)} \int_{\{x_a,\sigma_a\}} \widetilde{\rho}(x)\, |\psi_N|^2, \quad (3.23)$$

where $\omega_{\rm c}$ is the cyclotron frequency, $z = (x_1 + {\rm i}x_2)/\sqrt{2}l_B$ and $D_i = \partial_{\bar{z}_i} + z_i/2$, and hence we get

$$\mathcal{T}(x) := \langle W|\widehat{\mathcal{T}}(x)|W\rangle = \omega_{\rm c}\rho(x) \frac{\partial_z W^\dagger(x)\partial_{\bar{z}} W(x)}{W^\dagger(x)W(x)}. \quad (3.24)$$

Note that the kinetic energy density is zero when W is analytic.

From equations (3.17) and (3.20), we see that the density is a function of only $W^\dagger W$ while the spin density is determined by Z for a given density. Thus $W(x)$ can be provided with a physical meaning in terms of charge and spin densities.

3.3.1. Hydrodynamic limit. Evaluation of the correlation functions in the above nonlinear field theory is difficult, and hence we shall use the saddle-point approximation. In most calculations we shall further use the limit when $W_\sigma(x)$ varies slowly over the magnetic length scale. Since $W_\sigma(x)$ is related to density, as we shall see this is also the limit of small density fluctuations, and hence refer to it as the hydrodynamic limit.

In the hydrodynamic limit, the norm of the Laughlin wavefunctions is interpreted in terms of a classical 2D plasma problem. The same interpretation can be given here to the projected coherent state wavefunctions too, except that in this case the plasma density is coupled to an 'external field' $\ln(W^\dagger W)$.

In the large N limit, the Jacobian of the transformation is the entropy factor

$$J[\widetilde{\rho}] = \exp\left(\int_x (\widetilde{\rho}(x) - \widetilde{\rho}(x)\ln\widetilde{\rho}(x))\right). \quad (3.25)$$

Then the saddle-point equation is given by

$$\rho_{\rm cl}(x) = \bar{\rho} - \frac{1}{4\pi m}\nabla^2 \ln(W^\dagger(x)W(x)) + \frac{1}{4\pi m}\nabla^2 \ln\rho_{\rm cl}(x), \quad (3.26)$$

where the mean density $\bar{\rho} = (2\pi m l_B^2)^{-1}$, and the action at the saddle-point simplifies to

$$\mathcal{S}_{\rm cl}(\rho_{\rm cl}) = \int_x \left(-\rho_{\rm cl}(x) + 2\pi m\rho_{\rm cl}(x)\nabla^{-2}\rho_{\rm cl}(x)\right), \quad (3.27)$$

and, upon neglecting the non-Gaussian fluctuations, the correlation functions reduce to

$$\langle \widetilde{\rho}(x)\rangle_{\mathcal{S}} = \rho_{\rm cl}(x), \quad (3.28)$$

$$\langle \widetilde{\rho}(x)\widetilde{\rho}(y)\rangle_{\mathcal{S}} = \rho_{\rm cl}(x)\rho_{\rm cl}(x) + G(x,y), \quad (3.29)$$

where $[-4\pi m\nabla^{-2} + \rho_{\rm cl}^{-1}(x)]G(x,y) = \delta(x-y)$. Further assuming the density fluctuations to vary slowly gives the hydrodynamic results:

$$\rho(x) - \bar{\rho} \approx \frac{-1}{4\pi m}\nabla^2 \ln(W^\dagger(x)W(x)), \quad (3.30)$$

$$J_i(x) \approx \rho(x)[L_i^3(x) - e(A_i(x) - \alpha_i(x))], \quad (3.31)$$

where $\kappa\nabla \times \vec{\alpha}(x) := e\rho(x)$.

3.4. Charge and topological charge densities

We shall now find the relation between charge and topological charge when restricted to LLL. The topological charge density is defined as

$$q(x) := \frac{1}{8\pi}\epsilon_{ij}\hat{n}(x)\cdot\partial_i\hat{n}(x)\times\partial_j\hat{n}(x), \tag{3.32}$$

where $\hat{n}(x)$ is the local direction of spin polarization, $\vec{s}(x) = \frac{1}{2}\rho(x)\hat{n}(x)$. In terms of Z, it is given by

$$q(x) = \frac{1}{2\pi \mathrm{i}}\epsilon_{ij}\partial_i Z^\dagger(x)\partial_j Z(x). \tag{3.33}$$

As can be seen from equations (3.26) and (3.33), there is no *a priori* relation between the topological charge density and the electrical charge density independent of the state. This is also reflected in the fact that $W^\dagger W$ and Z are independent variables. However, if the LLL condition is imposed then the analyticity of W_σ relates the modulus and the phase of each component, and thus spin and charge get entangled. Using the analyticity condition, $\partial_i W_\sigma(x) = -\mathrm{i}\epsilon_{ij}\partial_j W(x)$, in equation (3.26) we get the relation,

$$\rho(x) = \bar{\rho} - \frac{1}{m}q(x) + \frac{1}{4\pi m}\nabla^2 \ln\rho(x). \tag{3.34}$$

In the hydrodynamic limit this relation reduces to

$$\rho(x) - \bar{\rho} = -\frac{1}{m}q(x). \tag{3.35}$$

Thus the topological charge density is proportional to the electrical charge density if and only if the LLL condition is satisfied. This relation will therefore not be true in the presence of Landau level mixing. The above relation (3.35) is known to hold for large-size quantum Hall skyrmions [13, 21].

When the densities are proportional, the total excess charge Q will, of course, be proportional to the total topological charge Q_{top}. However, the total charges could be proportional without the densities being so. We will now investigate this possibility. Integrating equation (3.30) over all space gives

$$Q := \int_x (\rho(x) - \bar{\rho}) = \frac{1}{4\pi m}\oint \mathrm{d}x^i\, \epsilon_{ij}\partial_j \ln(W^\dagger(x)W(x)), \tag{3.36}$$

where the contour is at infinity. If W is analytic at infinity, then the above equation becomes

$$Q = -\frac{1}{2\pi m}\oint \mathrm{d}x^i\, \frac{1}{2\mathrm{i}}(Z^\dagger(x)\partial_i Z(x) - \partial_i Z^\dagger(x)Z(x))$$
$$= -\frac{1}{m}Q_{\mathrm{top}}. \tag{3.37}$$

Thus if there is no Landau level mixing in the ground state, the total charge is always proportional to the topological charge. Note that Z and hence $q(x)$ are well defined only if $\rho(x)$ is non-zero everywhere. So these results are true only in such cases and will not hold for polarized vortices [7], where $\rho(x)$ vanishes at some point.

4. Coherent projected coherent states

We have seen that the projected coherent states could be labelled by a spinor field W. The computation of the expectation values of observables in terms of W, in turn, allowed us to further label these states by the physical observables, the density $\rho(x)$ and the normalized spinor $Z_\sigma(x)$. In this section, we will show that these states also satisfy the generalized coherent state properties [15] in $\mathcal{H}_{\mathrm{phy}}$, namely the resolution of identity and continuity of overlaps. This implies that the original electronic theory can be expressed with no redundancy in terms of bosonic field operators corresponding to $\rho(x)$ and $Z_\sigma(x)$.

4.1. Resolution of identity

The resolution of identity in $\mathcal{H}_{\mathrm{phy}}$ is obtained by the left and right action of the projection operator P on equation (2.12) and identifying P with the identity operator of $\mathcal{H}_{\mathrm{phy}}$. Thus we get

$$\int \mathcal{D}[a,\varphi]|a,\varphi\rangle_{\mathrm{pp}}\langle a,\varphi| = I, \tag{4.1}$$

and hence the projected coherent states form a basis for the physical Hilbert space. The left-hand side of the above equality, using equation (3.9) and making the change of variables

$$a_i(x), \varphi_\sigma(x) \to \Omega_{\mathrm{L}}(x), \Omega_{\mathrm{T}}(x), W_\sigma(x), \tag{4.2}$$

becomes

$$\int_{a,\varphi} |a,\varphi\rangle_{\mathrm{p}\,\mathrm{p}}\langle a,\varphi|$$
$$= \int \mathcal{D}[\Omega_{\mathrm{T}},\Omega_{\mathrm{L}},W]\, J[\Omega_{\mathrm{T}}]\,\mathcal{N}[W,\Omega_{\mathrm{T}}]|W\rangle\langle W|, \tag{4.3}$$

where the factor $J[\Omega_{\mathrm{T}}]$ is the Jacobian due to the change of variables $\varphi \to W$. Now integrating over Ω_{T} and Ω_{L} gives the resolution of identity, in terms of the gauge-invariant parameters W:

$$\int \mathcal{D}[W]\mathcal{G}[W]\,|W\rangle\,\langle W| = I, \tag{4.4}$$

where the measure consists of

$$\mathcal{D}[W] = \prod_{x,\sigma}\frac{\mathrm{d}W_\sigma(x)\,\mathrm{d}\bar{W}_\sigma(x)}{2\pi\mathrm{i}}, \tag{4.5}$$

and

$$\mathcal{G}[W] := \mathcal{Z}[W^\dagger W]\int \mathcal{D}[\Omega_{\mathrm{T}},\Omega_{\mathrm{L}}]J[\Omega_{\mathrm{T}}]\,|C[W,\Omega_{\mathrm{T}}]|^2, \tag{4.6}$$

which in the hydrodynamic limit, as briefed in appendix D, reduces to

$$\mathcal{G}[W] \approx c_1\prod_x \frac{1}{[W^\dagger(x)W(x)]^2}, \tag{4.7}$$

where c_1 is a constant.

In terms of density and spin variables the resolution of identity is obtained by the transformation $W_\sigma(x) \rightarrow \{\rho(x), Z_\sigma(x)\}$:

$$\int \mathcal{D}[\rho, Z]\, |\rho, Z\rangle\, \langle\rho, Z| = I \tag{4.8}$$

where the measure now becomes

$$\mathcal{D}[\rho, Z] = c_1' \prod_x \mathrm{d}\rho(x)\ \sin^2\theta(x) \sin\phi(x)\, \mathrm{d}\theta(x)\, \mathrm{d}\phi(x) \times \mathrm{d}\psi(x), \tag{4.9}$$

when Z is parametrized as

$$Z = \mathrm{e}^{(\mathrm{i}/2)\psi} \begin{pmatrix} \cos\frac{\theta}{2}\, \mathrm{e}^{(\mathrm{i}/2)\phi} \\ \sin\frac{\theta}{2}\, \mathrm{e}^{(\mathrm{i}/2)\phi} \end{pmatrix}. \tag{4.10}$$

4.2. Overlaps

The overlap of the projected coherent states, $|a_1, \varphi_1\rangle_\mathrm{p}$ and $|a_2, \varphi_2\rangle_\mathrm{p}$, can be written as the partition function with an 'external field' $\ln W_1^\dagger W_2$:

$$\begin{aligned} {}_\mathrm{p}\langle a_1, \varphi_1 | a_2, \varphi_2\rangle_\mathrm{p} &= \sum_N \frac{1}{N!} \prod_{a=1}^{N} \int_{x_a, \sigma_a} \overline{\psi}_N(\{x_a, \sigma_a\}; 1)\psi_N(\{x_a, \sigma_a\}; 2) \\ &= C[a_1, \varphi_1]C[a_2, \varphi_2]\mathcal{Z}[W_1^\dagger W_2]. \end{aligned} \tag{4.11}$$

Hence, the overlap of their normalized states, $|W_1\rangle$ and $|W_2\rangle$, is given by

$$\langle W_1 | W_2\rangle = \frac{\mathcal{Z}[W_1^\dagger W_2]}{\sqrt{\mathcal{Z}[W_1^\dagger W_1]\mathcal{Z}[W_2^\dagger W_2]}} \tag{4.12}$$

and, in the hydrodynamic limit, reduces to

$$\langle W_1 | W_2\rangle \approx \exp\left(-\left(\mathcal{S}_\mathrm{cl}(\rho_{12}) - \tfrac{1}{2}\mathcal{S}_\mathrm{cl}(\rho_1) - \tfrac{1}{2}\mathcal{S}_\mathrm{cl}(\rho_2)\right)\right), \tag{4.13}$$

where $\mathcal{S}_\mathrm{cl}$ are the saddle-point actions, as given in equation (3.27), with the densities

$$\begin{aligned} \rho_a(x) &= \overline{\rho} - \frac{1}{4\pi m}\nabla^2 \ln W_a^\dagger W_a, \\ \rho_{12}(x) &= \overline{\rho} - \frac{1}{4\pi m}\nabla^2 \ln W_1^\dagger W_2 \\ &= \frac{\rho_1 + \rho_2}{2} - \frac{1}{4\pi m}\nabla^2 \ln Z_1^\dagger Z_2, \end{aligned} \tag{4.14}$$

for $a = 1$ and 2. Using the above expressions we get the overlap in terms of physical observables:

$$\langle W_1 | W_2\rangle = \exp\left(\int_x \left(\frac{m\pi}{2}(\rho_1 - \rho_2)\nabla^2(\rho_1 - \rho_2) + \frac{\rho_1 + \rho_2}{2} \ln Z_1^\dagger Z_2 - \frac{1}{8\pi m} \ln Z_1^\dagger Z_2 \nabla^2 \ln Z_1^\dagger Z_2\right)\right). \tag{4.15}$$

In terms of spin polarizations, $Z_1^\dagger Z_2 = ((1 + \hat{n}_1 \cdot \hat{n}_2)/2)^{1/2} \exp(\mathrm{i}\Phi(\hat{n}_1, \hat{n}_2)/2)$, where $\Phi(\hat{n}_1, \hat{n}_2)$ is the solid angle subtended by the geodesic triangle with $\hat{n}_1, \hat{n}_2$ and some third point on the unit sphere as vertices. Note that the overlap smoothly goes to 1 as $\{\rho_1, Z_1\} \rightarrow \{\rho_2, Z_2\}$.

The overlap of the neighbouring states $|W\rangle$ and $|W + \epsilon\partial_t W\rangle$ can be evaluated to $O(\epsilon)$ by using the identity

$$\mathcal{Z}[\Lambda + \delta\Lambda] = \mathcal{Z}[\Lambda]\left(1 + \int_x \langle\widetilde{\rho}(x)\rangle_S \frac{\delta\Lambda(x)}{\Lambda(x)} + O\left(\delta\Lambda^2\right)\right), \tag{4.16}$$

in equation (4.12), and we get

$$\langle W + \epsilon\partial_t W | W\rangle = 1 - \mathrm{i}\epsilon \int_x \rho(x) L_t^3(x) + O(\epsilon^2) \tag{4.17}$$

where $L_\mu^3 := (Z^\dagger\partial_\mu Z - h.c)/2\mathrm{i}$ for $\mu = t$, 1 and 2.

If we now impose the LLL condition, and hence use equation (3.35), the theory can then be expressed in terms of spin fluctuations alone. Note that in terms of L_i^3 the topological charge density $q(x) = (2\pi)^{-1}\epsilon_{ij}\partial_i L_j^3(x)$. The expression for the overlap in equation (4.17) then becomes

$$\langle W + \epsilon\partial_t W | W\rangle = 1 - \mathrm{i}\epsilon\overline{\rho} \int_x L_t^3(x) + \mathrm{i}\epsilon\frac{1}{4\pi m} \int_x \epsilon_{\mu\nu\lambda} L_\mu^3(x)\partial_\nu L_\lambda^3(x). \tag{4.18}$$

The second term on the right-hand side of the above equation is the solid angle term, while the last term is the Hopf term.

5. Classical limit

The path-integral representation in terms of the physical variables follows from the resolution of the identity (4.8) and (4.9), and the overlaps (4.17) and (4.18). Thus we have a classical theory defined on the manifold parametrized by $\{\rho, Z\}$ with the symplectic structure provided by the overlap of the neighbouring states (4.17). This theory when restricted to the lowest Landau level becomes, from equation (4.18), a nonlinear sigma model (NLSM) with a Hopf term in the action.

We shall now show that for a large skyrmionic configuration the theory becomes classical. Consider the set of states corresponding to configurations characterized by a single size parameter, λ. We parametrize them as

$$\begin{aligned} \rho^\lambda(x) &= \bar{\rho} + \frac{1}{\lambda^2}\Delta\rho\left(\frac{x}{\lambda}\right), \\ Z^\lambda(x) &= Z\left(\frac{x}{\lambda}\right). \end{aligned} \tag{5.1}$$

Substituting $\rho_1^\lambda(x)$, $Z_{1\sigma}^\lambda(x)$ and $\rho_2^\lambda(x)$, $Z_{2\sigma}^\lambda(x)$ in equation (4.15) and changing the variable $x \rightarrow \lambda x$ we get to the leading order in λ

$$\langle W_1 | W_2\rangle = \exp\left(\frac{1}{2}\lambda^2\overline{\rho} \int_x \ln\left(\frac{1 + \hat{n}_1 \cdot \hat{n}_2}{2}\right) + \mathrm{i}\Phi(\hat{n}_1, \hat{n}_2)\right) \tag{5.2}$$

and thus, for $W_1 \neq W_2$

$$\lim_{\lambda\rightarrow\infty} \langle W_1 | W_2\rangle \rightarrow 0. \tag{5.3}$$

The coherent states thus become orthogonal when $\lambda \rightarrow \infty$ and the off-diagonal matrix elements of the observables in this basis vanish. Since equation (5.2) is independent of ρ, the same relation holds even when restricted to LLL. Hence the set of states corresponding to a system of skyrmions will behave classically in the limit of the skyrmion sizes tending to infinity.

6. Conclusion: application to quantum Hall skyrmions

In this section, we summarize and apply the above formalism to some aspects of LLL quantum Hall systems.

In this work, we have developed a bosonization scheme using field coherent states. We started with the composite boson Hilbert space $\mathcal{H}_{\rm CB}$ with a coherent state basis. The physical subspace of this Hilbert space $\mathcal{H}_{\rm phy}$ is then mapped to the electronic Hilbert space $\mathcal{H}_{\rm el}$. The coherent states projected onto the physical subspace are shown to carry coherent state properties, form a basis for $\mathcal{H}_{\rm phy}$ and could be parametrized by a spinor field $W(x)$.

The matrix elements of the operators in this basis are expressed as the correlation functions of a field theory, which are then evaluated in the hydrodynamic limit. The LLL condition is found to be equivalent to the condition that $W_\sigma(x)$ are analytic functions. When restricted to LLL we show that the charge and topological charge get tied up. We also showed that the condition for the total charge density to be proportional to the topological charge is weaker. It only requires $W(x)$ to be analytic at infinity; in other words, that the ground state does not have Landau level mixing.

We also interpret W in terms of density and spin density. A path-integral representation of the evolution operator in terms of the physical variables is obtained. The set of states corresponding to classical configurations, characterized by a length scale λ, become orthogonal in the limit of $\lambda \to \infty$. This implies that the corresponding classical theory becomes exact in this limit.

This formalism is particularly suitable for semiclassical study of $(2+1)$-dimensional electronic systems with quantum constraints, like LLL restriction. First we can easily find the semiclassical ground state of quantum Hall systems. In the classical limit, the expectation value of the Hamiltonian in an arbitrary state, $|\psi\rangle = \sum_W A_W|W\rangle$, becomes $\langle\psi|H|\psi\rangle \to \sum_W |A_W|^2\langle W|H|W\rangle \geqslant \langle W_{\rm m}|H|W_{\rm m}\rangle$, where $W_{\rm m}$ is the configuration corresponding to the minimum expectation value of energy. A constant W leads to the minimum energy with $\rho = \overline{\rho}$. Hence we find that the semiclassical ground state, whether or not the system is restricted to LLL, is the Laughlin state at filling fractions $\nu = 1/m$.

Quantum Hall systems support spin-textured quasiparticle excitations called skyrmions, which have been extensively studied both experimentally [22–26] and theoretically [27, 13, 21]. These quasihole (particle) excitations are assumed to be described by the classical solutions of NLSM. The electrical charge density is assumed to be equal to the topological charge density: $\rho - \overline{\rho} = -q/m$. Near $\nu = 1$, the NLSM energy functional and the relation between the topological charge density and electrical charge density has been derived by many in the LLL, long-wavelength approximation [13, 28, 29]. The question, about the regime of validity of this model and the limit when the classical approximation is exact, was also numerically addressed previously [30] and the energetics of large scale skyrmions suggest that the classical approximation may be exact in the long-wavelength limit.

In our formalism, starting from the microscopic theory we could derive NLSM with the Hopf term as the phase space of skyrmions for $\nu = 1/m$. We could also answer analytically the following questions. When is the topological charge proportional to the electric charge? When are the corresponding densities proportional to each other? What is the limit in which the classical approximation is exact? How can the LLL condition be imposed in the classical theory?

The large-size skyrmions have the property that the charge density is proportional to topological charge density. Hence the projected coherent states $|W\rangle$, which have this property, are good candidates to describe the skyrmionic excitations. By adding the corrections to this relation, namely equation (3.34), we can also describe small skyrmions classically. Further a semiclassical description can be provided with the corresponding coherent states. The classical limit suggests that NLSM is an exact description when the size of the skyrmions tends to infinity.

This formalism has the advantage of easily incorporating the effects of Landau level mixing. This is because the LLL constraint has been imposed only after obtaining the bosonic theory for a generic $(2+1)$-dimensional fermionic system. We saw that the relation between the topological charge and the electric charge, unlike that of their corresponding densities, remains unaffected by Landau level mixing if only the ground state is restricted to LLL. Finally, it may also be suitable to study the system in a slowly varying external potential.

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Appendix A. The path-integral representation

In this appendix, we derive the path-integral representation of the partition function by the usual procedure of splitting the time interval t into N segments of length ϵ. At each intermediate step inserting equation (4.1), namely the resolution of identity of $\mathcal{H}_{\rm phy}$, and then taking the limit, $\epsilon \to 0$, $N \to \infty$ keeping $t = \epsilon N$ fixed, gives

$$Z = \mathrm{Tr}\,\mathrm{e}^{-\mathrm{i}Ht} = \mathrm{Tr}[\mathrm{e}^{-\mathrm{i}\epsilon H}]^N = \int \mathcal{D}[a,\varphi] \prod_{n=0}^{N} \langle a_{n+1}, \varphi_{n+1}| P\mathrm{e}^{-\mathrm{i}\epsilon H} P |a_n, \varphi_n\rangle, \qquad (\mathrm{A.1})$$

where the measure $\mathcal{D}[a,\varphi] = \prod_n \mathcal{D}[a_n,\varphi_n]$, and $(a_{N+1},\varphi_{N+1}) = (a_0,\varphi_0)$. This measure is also gauge-invariant, while under a gauge transformation

$$|a,\varphi\rangle\langle a,\varphi| \to |a - \nabla\beta, \varphi\mathrm{e}^{-\mathrm{i}e\beta}\rangle\langle a - \nabla\beta, \varphi\mathrm{e}^{-\mathrm{i}e\beta}|, \qquad (\mathrm{A.2})$$

where $\beta(x)$ is an arbitrary function. Hence in the above expression (A.1) we replace a_n and φ_n by $a_n - \nabla\beta_n$ and $\varphi_n \exp(-\mathrm{i}e\beta_n)$, respectively, for all n. Since the gauge-invariant Hamiltonian operator H commutes with P and $P^2 = P$, we make the projection operator act only on the *kets* and not on the *bras*, for all n. These steps result in

$$Z = \int \mathcal{D}[a, \varphi, \Omega] \prod_{n=0}^{N} \mathrm{e}^{\mathrm{i}(\kappa/2)\int_x \vec{a}_n(x)\times\nabla\Omega_n(x)} \times \langle a_{n+1} - \nabla\beta_{n+1}, \varphi_{n+1}\mathrm{e}^{-\mathrm{i}\,e\beta_{n+1}}|\mathrm{e}^{-\mathrm{i}\epsilon H}|a_n - \nabla\Omega_n - \nabla\beta_n, \varphi_n\mathrm{e}^{-\mathrm{i}\,e(\Omega_n+\beta_n)}\rangle \tag{A.3}$$

where $\mathcal{D}[a, \varphi, \Omega] = \mathcal{D}[a, \varphi]\prod_n (V_\Omega)^{-1}\int_{\Omega_n}$. Now choose the function $\beta_{n+1}(x) = \Omega_{n+1}(x) + \beta_n(x)$ and expand the functions $a_{n+1} = a_n + \epsilon\dot{a}_n$, $\Omega_{n+1} = \Omega_n + \epsilon\dot{\Omega}_n$ and $\varphi_{n+1} = \varphi_n + \epsilon\dot{\varphi}_n$, to $O(\epsilon)$. Then the above expression to $O(\epsilon)$ becomes

$$Z = \int \mathcal{D}[a, \varphi, \Omega] \prod_{n=0}^{N} \mathrm{e}^{\mathrm{i}(\kappa/2)\int_x \vec{a}_n(x)\times\nabla\Omega_n(x)} \times \langle a'_n + \epsilon\delta a'_n, \varphi'_n + \epsilon\delta\varphi'_n | a'_n, \varphi'_n\rangle \times \left(1 - \mathrm{i}\epsilon\frac{\langle a'_n, \varphi'_n|H|a'_n, \varphi'_n\rangle}{\langle a'_n, \varphi'_n|a'_n, \varphi'_n\rangle}\right), \tag{A.4}$$

where $a'_n = a_n - \nabla\Omega_n - \nabla\beta_n$, $\varphi'_n = \varphi_n \exp(-\mathrm{i}e(\Omega_n + \beta_n))$, $\delta a'_n = \dot{a}_n - \nabla\dot{\Omega}_n$ and $\delta\varphi'_n = (\dot{\varphi}_n - \mathrm{i}e\varphi_n\dot{\Omega}_n)\exp(-\mathrm{i}e(\beta_n + \Omega_n))$. The first two terms in the above integrand, to $O(\epsilon)$, simplify to

$$\mathrm{e}^{\mathrm{i}(\kappa/2)\int_x \vec{a}_n(x)\times\nabla\Omega_n(x)}\langle a'_n + \epsilon\delta a'_n, \varphi'_n + \epsilon\delta\varphi'_n|a'_n, \varphi'_n\rangle = 1 + \mathrm{i}\epsilon\int_x\left(-\frac{\kappa}{2}\epsilon_{\mu\nu\lambda}a_n^\mu\partial^\nu a_n^\lambda + ea_{0n}\varphi_n^\dagger\varphi_n - \frac{\mathrm{i}}{2}(\dot{\varphi}_n^\dagger\varphi_n - \varphi_n^\dagger\dot{\varphi}_n)\right), \tag{A.5}$$

where the indices μ, ν, λ run over 1, 2 and 3, $\dot{\Omega}_n$ is given a new notation a_{0n}, and the x dependence of a_n^μ, φ_n and $\varphi_n^\dagger$ is not explicitly displayed. The matrix elements in the last term of the integrand $H(a'_n, \varphi'_n) = H(a_n, \varphi_n)$, since H is gauge-invariant. Now substituting equation (A.5) in equation (A.4) and taking the limit $\epsilon \to 0$ finally results in the familiar Chern–Simons path integral, as given in equation (2.19).

Appendix B. The wavefunctions

In this appendix, we evaluate the projected coherent state wavefunctions. The action of projection operator P and of the electron operator $c_{\sigma_a}(x_a)$ on the coherent states is easily obtainable and is given by

$$P|a, \varphi\rangle = \frac{1}{V_{\mathrm{G}}}\int_\Omega \mathrm{e}^{\mathrm{i}(\kappa/2)\int_x \vec{a}(x)\times\nabla\Omega(x)}\,\left|a - \partial\Omega, \varphi\mathrm{e}^{-\mathrm{i}\,e\Omega}\right\rangle, \tag{B.1}$$

and

$$c_{\sigma_a}(x_a)|a, \varphi\rangle = \varphi_{\sigma_a}(x_a)\,\mathrm{e}^{\mathrm{i}(\kappa/2)\int_x \vec{a}(x)\times\vec{a}^v_{x_a}(x)}\,\left|a - a^v_{x_a}, \varphi\mathrm{e}^{\mathrm{i}m\theta_{x_a}}\right\rangle, \tag{B.2}$$

respectively. Here $\varphi\exp(\mathrm{i}m\theta_{x_a})$ denotes the configuration $\varphi(x)\exp(\mathrm{i}m\theta(x_a - x))$ and $\partial\Omega$ is the complex variable notation of $\nabla\Omega$. Using the above expressions we get

$$\prod_{a=1}^{N} c_{\sigma_a}(x_a)P|a, \varphi\rangle = \frac{1}{V_{\mathrm{G}}}\int_\Omega \mathrm{e}^{\mathrm{i}(\kappa/2)\int_x \vec{a}\times\nabla\Omega}\prod_{a=1}^{N}\varphi_{\sigma_a}(x_a)\mathrm{e}^{-\mathrm{i}\,e\Omega(x_a)}\prod_{b=1}^{a-1}\mathrm{e}^{\mathrm{i}m\theta(x_a - x_b)} \times \mathrm{e}^{\mathrm{i}(\kappa/2)\int_x(\vec{a}-\nabla\Omega)\times\vec{A}^v}\,\left|a - \partial\Omega - A^v, \varphi\mathrm{e}^{-\mathrm{i}\,e(\Omega+\Theta)}\right\rangle, \tag{B.3}$$

and, for notational convenience, $A^v = \sum_a a^v_{x_a}$ and $\Theta = \sum_a \theta_a$, are used, and the x dependence in the integrals is not shown explicitly. In obtaining the above expression, further we have dropped the $\int_x \vec{a}^v_{x_a} \times \vec{a}^v_{x_b}$ terms, for we shall choose the vectors $\vec{a}^v_{x_a}$ such that their longitudinal components vanish. We also write $\vec{a}(x)$ as

$$a_i(x) = \epsilon_{ij}\partial_j\Omega_{\mathrm{T}}(x) + \partial_i\Omega_{\mathrm{L}}(x), \tag{B.4}$$

and often denote this decomposition by $\vec{a}(x) = \vec{a}_{\mathrm{T}}(x) + \vec{a}_{\mathrm{L}}(x)$. Now making the change of variable, $\Omega \to \Omega + \Omega_{\mathrm{L}}$, in equation (B.3) and taking the overlap with the vacuum state gives

$$\psi_N(\{x_a, \sigma_a\}) = \frac{1}{V_{\mathrm{G}}}\mathrm{e}^{\mathrm{i}(\kappa/2)\int_x \vec{a}_{\mathrm{T}}\times\vec{a}_{\mathrm{L}}}\prod_{a=1}^{N}\varphi_{\sigma_a}(x_a)\mathrm{e}^{-\mathrm{i}\,e\Omega_{\mathrm{L}}(x_a)} \times \prod_{b=1}^{a-1}\mathrm{e}^{\mathrm{i}m\theta(x_a-x_b)}\int_\Omega \mathrm{e}^{(\mathrm{i}/2)\int_x\Omega(\kappa\nabla\times\vec{a} - \mathrm{e}\sum_a\delta(x-x_a))} \times \langle 0|a_{\mathrm{T}} - \partial\Omega - A^v, \varphi\rangle. \tag{B.5}$$

Here we have made use of the relation $\kappa\int_x \nabla\Omega \times \vec{a}^v_{x_a} = -\mathrm{e}\Omega(x_a)$ and the identities $\int \vec{u}_{\mathrm{T}}\times\vec{v}_{\mathrm{T}} = \int \vec{u}_{\mathrm{L}}\times\vec{v}_{\mathrm{L}} = \int \vec{u}_{\mathrm{T}}\cdot\vec{v}_{\mathrm{L}}$, which hold for the arbitrary vectors $\vec{u}$ and $\vec{v}$. Note that the wavefunction vanishes unless the total flux equals the total number of particles, for the zero momentum mode of the Ω integral gives the constraint

$$\kappa\int_x \nabla\times\vec{a}(x) = eN. \tag{B.6}$$

The last term in equation (B.5) is explicitly given by

$$\langle 0|a_{\mathrm{T}} - \partial\Omega - A^v, \varphi\rangle = \mathrm{e}^{-(1/2)\int_x\varphi^\dagger\varphi}\mathrm{e}^{-(\kappa/4)\int_x\nabla\Omega\cdot\nabla\Omega}\mathrm{e}^{-(\kappa/4)\int_x\left|\vec{a}_{\mathrm{T}} - \vec{A}^v\right|^2}, \tag{B.7}$$

and hence the Ω-dependent part when integrated becomes

$$\int_\Omega \mathrm{e}^{-(\kappa/4)\int_x\nabla\Omega\cdot\nabla\Omega}\mathrm{e}^{\mathrm{i}(\kappa/2)\int_x\Omega\nabla\times(\vec{a}_{\mathrm{T}} - \vec{A}^v)} = \mathrm{e}^{-(\kappa/4)\int_x\left|\vec{a}_{\mathrm{T}} - \vec{A}^v\right|^2}\int_\Omega \mathrm{e}^{-(\kappa/4)\int_x\nabla\Omega\cdot\nabla\Omega}. \tag{B.8}$$

The integral in the first term on the RHS is straightforward to evaluate and is obtained as

$$\kappa\int_x\left|\vec{a}_{\mathrm{T}} - \vec{A}^v\right|^2 = \kappa\int_x|\vec{a}_{\mathrm{T}}|^2 - 2e\sum_a\Omega_{\mathrm{T}}(x_a) - 2m\sum_{a>b}\ln|x_a - x_b|, \tag{B.9}$$

up to a (infinite) constant from $a = b$ terms. Now using the expressions (B.7), (B.8) and (B.9) in equation (B.5), we get

$$\psi_N(\{x_a, \sigma_a\}) = C[\vec{a}, \varphi]\prod_{a=1}^{N}\varphi_{\sigma_a}(x_a)\mathrm{e}^{e\Omega_{\mathrm{T}}(x_a) - \mathrm{i}\,e\Omega_{\mathrm{L}}(x_a)} \times \prod_{a>b}^{N,N}(z_a - z_b)^m, \tag{B.10}$$

where $(z_a - z_b) = |x_a - x_b|\exp(\mathrm{i}\theta(x_a - x_b))$. The factor $C[a, \varphi]$ contains terms that are independent of the coordinates $\{x_a, \sigma_a\}$:

$$C[\vec{a}, \varphi] = c_0\mathrm{e}^{\mathrm{i}(\kappa/2)\int_x\vec{a}_{\mathrm{T}}\times\vec{a}_{\mathrm{L}}}\mathrm{e}^{-(\kappa/2)\int_x|\vec{a}_{\mathrm{T}}|^2}\mathrm{e}^{-(1/2)\int_x\varphi^\dagger\varphi}, \tag{B.11}$$

where c_0 is a constant, and the phase can be made to vanish by a gauge choice. The expression (B.10) is then finally rewritten as given in equation (3.3).

Appendix C. Projection operator and vertex operators

Using equations (B.1) and (2.13) we get

$$\langle a,\varphi|P|a,\varphi\rangle = \frac{1}{V_G} e^{-\int_x \varphi^\dagger\varphi} \int_\Omega e^{-(\kappa/4)\int_x (\nabla\Omega)^2} \times e^{i\kappa\int_x \Omega\nabla\times\vec{a}} e^{\int_x \varphi^\dagger\varphi \exp(-ie\Omega)}. \quad (C.1)$$

If $\kappa\int_x \nabla\times\vec{a} = eN$ then the above expression, upon Taylor expanding the last exponential term, reduces to the following correlation function of vertex operators:

$$\langle a,\varphi|P|a,\varphi\rangle = \frac{1}{V_G} e^{-\int_x \varphi^\dagger\varphi} \frac{1}{N!}\prod_{i=1}^{N}\int_{x_i}\varphi^\dagger(x_i)\varphi(x_i) \times \int_\Omega e^{-(\kappa/4)\int_x(\nabla\Omega)^2} e^{i\kappa\int_x \Omega\nabla\times\vec{a}} \prod_{i=1}^{N} e^{-ie\Omega(x_i)}. \quad (C.2)$$

Appendix D. Gauge-invariant measure

The Jacobian of the transformation, $\varphi \to W = \varphi\exp(e(\Omega_T - \widetilde{\Omega}_T - i\Omega_L))$, is $J[\Omega_T] = \exp(-4e(\Omega_T - \widetilde{\Omega}_T - i\Omega_L))$. Therefore the integral

$$\int_{\Omega_T,\Omega_L} J[\Omega_T]|C|^2 = \int_{\Omega_T}\prod_x e^{-4e\Omega_T(x)} e^{-\kappa\int_x(\nabla\Omega_T)^2} \times e^{-\int_x W^\dagger W e^{-2e(\Omega_T-\widetilde{\Omega}_T)}}. \quad (D.1)$$

We now do the Ω_T integration about the saddle-point which, to $O(A_\epsilon^2)$, is given by

$$\Omega_T(x) = \widetilde{\Omega}_T(x) + \frac{1}{2e}\ln W^\dagger(x)W(x) - \frac{1}{2e}\ln\frac{2}{A_\epsilon} + \frac{A_\epsilon\kappa}{4e^2}\nabla^2\Omega_T(x), \quad (D.2)$$

where A_ϵ is the area of the unit cell of the lattice used for regularizing the measure. The last term in the above equation can be neglected if $2e\ln W^\dagger W \gg A_\epsilon\kappa\nabla^2\ln W^\dagger W$. This condition holds good, even without invoking the hydrodynamic limit, when $A_\epsilon \to 0$. Now neglecting the non-Gaussian fluctuations, we get

$$\int_{\Omega_T,\Omega_L} J[\Omega_T]|C|^2 = c_1\exp\left(-\int_x \rho + 2\pi m\int_x \rho\nabla^{-2}\rho\right)\prod_x \left(W^\dagger(x)W(x)\right)^{-2}. \quad (D.3)$$

Note that the factor in the exponential is the action $\mathcal{S}_{cl}(\rho_{cl})$ at the saddle-point.

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